Final Exam

Exercise 1 Let X_1 and X_2 be a random sample of size 2 from the exponential distribution with pdf

$$f(x) = 2e^{-2x} \quad 0 < x < +\infty$$

a. find $P(0.5 < X_1 < 1 \cap 0.4 < X_2 < 0.8)$

b. find $E(3X_1^2 X_2)$

Exercise 2 Let $X \hookrightarrow b(10, \frac{1}{3})$ and $Y \hookrightarrow b(15, \frac{1}{3})$ be two independent binomial distributions. Let $Z = 25 - X - Y$. Find $P(Z \geq 2)$.

Exercise 3 Let X and Y have joint pdf

$$f(x, y) = 2x(x - y) \qquad 0 \leq x \leq 1, \ -x \leq y \leq x$$

a. find the joint pdf of $U = X$ and $V = X - Y$.

b. find the marginal distributions of U and V. Are U and V independent?

Exercise 4 Suppose that the length of life in hours of a light bulb manufactured by company A is $\mathcal{N}(800, 14400)$ and the length of life in hours of a light bulb manufactured by company B is $\mathcal{N}(850, 2500)$. One bulb is selected from each company and is burned until death.

a. find the probability that length life of the bulb from company A exceeds the length of life of the bulb from company B by at least 15 hours.

b. find the probability that at least one of the bulbs lives for at least 920 hours.

Exercise 5 Let $\overline{X} = \sum_{i=1}^{n} X_i$, where X_i are i.i.d. with $f(x) = \frac{1}{\theta}e^{-x/\theta}$, $0 < x < +\infty$. Use generating functions to find the distribution of $Y = (2n/\theta)\overline{X}$.

Exercise 6 Fifty numbers are rounded off to the nearest integer and then summed. If the individual round-off errors are uniformly distributed over the interval $(-1/2; 1/2)$, what is the probability that the resultant sum differs from the exact sum by more than 3 ?

(hint: remember the gambler!)